Lecture 4-3: Inference and Interpretation

Goal: Explain inference with the OLS slope and how to interpret it: tests, confidence intervals, large-sample inference, and log models.

- Inference with OLS
  - OLS assumption VI (normal errors)
  - Hypothesis testing and confidence intervals
  - Large sample properties
- Interpreting OLS coefficients
  - Descriptive, predictive, causal uses
  - Best linear predictor
  - Predictive differences
  - Practical interpretations
  - Substantive vs. statistical significance
  - Incorporating nonlinearities with log transformations
  - Common model forms (level-level, log-level, etc.)

Worksheet

1. (*Consistency of OLS*) If $\text{Cov}(X, u) = 0$ and $\text{Var}(X) > 0$, show $\hat{\beta}_1 \xrightarrow{p} \beta_1$.

2. (*Asymptotic normality and CI for* β_1) Under Assumptions I–V, prove that

$$\frac{\hat{\beta}_1 - \beta_1}{\widehat{SE}(\hat{\beta}_1)} \xrightarrow{d} N(0,1), \qquad \widehat{SE}(\hat{\beta}_1) = \frac{\hat{\sigma}_u}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}, \quad \hat{\sigma}_u^2 = \frac{1}{n}\sum_{i=1}^n \hat{u}_i^2,$$

and derive the large–sample $100(1-\alpha)\%$ confidence interval for β_1.

3. (*Log–level model with a binary regressor*) Consider $\log(Y) = \beta_0 + \beta_1 D + u$ with $D \in \{0, 1\}$, assuming $D \perp u$.

   (a) Derive the exact percent change in $E[Y \mid D]$ when D moves from 0 to 1.

   (b) Explain why the rule "$100 \cdot \beta_1\%$" is generally invalid here, and relate the sign and magnitude of the approximation error to β_1.

   (c) For $\beta_1 = 0.30$, compute the exact percent change and the approximation error in percentage points.

   (d) Find the value of $\beta_1 > 0$ above which the absolute approximation error exceeds 1 percentage point.