Lecture 5-4: Interaction Terms and Polynomials

Goal: Explore interaction terms with continuous variables, their common pitfalls, and polynomial regressions.

- Interaction terms
  - Why use interaction terms
  - One dummy variable and one continuous variable
  - Two continuous variables
  - Common pitfalls
    - Omitted lower order terms
    - Misinterpreting coefficients
    - Ignoring covariance in standard errors
    - Excessive extrapolation
    - False linearity assumption
- Polynomials
  - Polynomial regression as interactions
  - Marginal effects and interpretations
  - Higher order polynomials

Worksheet

1. Let X_{court} be high court independence (0–4) and $Z_{dem} \in \{0, 1\}$ indicate democracy. Estimate

$$Y = \beta_0 + \beta_1 X_{court} + \beta_2 Z_{dem} + \beta_3 (X_{court} \cdot Z_{dem}) + u.$$

   (a) Derive the conditional marginal effect of court independence at regime type $Z = z$ and the conditional marginal effect of democracy at court independence $X = x$.

   (b) Using the delta method, give $\operatorname{Var}\big(\widehat{\mathrm{ME}}_{court}(z)\big)$ and $\operatorname{Var}\big(\widehat{\mathrm{ME}}_{dem}(x)\big)$, where $\widehat{\mathrm{ME}}_{court}(z) = \hat{\beta}_1 + \hat{\beta}_3 z$ and $\widehat{\mathrm{ME}}_{dem}(x) = \hat{\beta}_2 + \hat{\beta}_3 x$.

   (c) Explain why omitting the covariance term biases the standard errors.

2. Consider $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + u$ (a special case of continuous interactions).

   (a) Derive the marginal effect of X and the location of the turning point. State how the sign of β_2 determines shape.

   (b) Using the delta method, derive the standard error of the estimated turning point $\hat{\tau} = -\hat{\beta}_1/(2\hat{\beta}_2)$. (Hint: the delta method linearizes, $\hat{\tau} - \tau \approx \frac{\partial \tau}{\partial \beta_1}(\hat{\beta}_1 - \beta_1) + \frac{\partial \tau}{\partial \beta_2}(\hat{\beta}_2 - \beta_2)$; then use the variance of a sum.)

   (c) Give the variance of the marginal effect at $X = x$.